CALCULATION OF DROPLET CONDENSATION

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A solution of the nonstationary problem of droplet growth is presented together with results of a study of the effect of edge wetting angle on increase in droplet size and amount of heat dissipated by the droplet.

The problem of droplet growth is a major one in methods of calculating heat exchangers which operate in droplet condensation regimes. Usually [1-4] the problem is solved in a quasistationary formulation, dropping the local term in the thermal conductivity equation and assuming, as a rule, that boundary conditions are stationary. In [1] this approach was justified on the grounds that the problem of droplet growth belongs to the class of so-called Stefan problems, methods for the solution of which have not been sufficiently developed.

In [1, 3] the temperature distribution was obtained in the form of a diverging series of Legendre polynomials. In [5] the quasistationary and nonstationary problems were solved with the aid of asymptotic representations, which limits the applicability to low limiting wetting angles, which are void of interest in those cases where reliable droplet condensation must be provided. At the same time, numerical methods permit consideration of the local term in the thermal conductivity equation, the change in the boundaries of the integration region with time, and the dependence of boundary conditions upon this change. Below we will present a solution of this problem with some results of numerical calculations.

We will consider a droplet on a plane wall (Fig. 1a), the temperature of which T_W is constant and given. The boundary conditions on the droplet free surface have the form

$$\alpha(\vartheta_v - \vartheta) + r\beta_p(p_v - p_{ls}) = -\lambda_l \frac{\partial \vartheta}{\partial \zeta}. \tag{1}$$

Here $\vartheta = T - T_W$, where T is the droplet temperature, a function of two coordinates, the radial ζ and meridional φ (temperature is assumed constant along longitude). The coefficient of convective heat transfer from the moving vapor to the droplet α can be found, e.g., from the expression [6]

$$\mathrm{Nu} = 2 + 0.03\,\mathrm{Pr}^{0.33}\mathrm{Re}^{0.54} + 0.35\mathrm{Pr}^{0.36}\mathrm{Re}^{0.58} \tag{2}$$

or [7]

$$\mathrm{Nu} = 0.41\mathrm{Re}_{\mathrm{nar}}^{0.6}\mathrm{Pr}^{0.33}. \tag{3}$$

Following Berman [7], we define the mass-transfer coefficient from the formula

$$\mathrm{Nu}_D = 0.55\mathrm{Nu}_{Dl}\Pi_g^{-0.35}\varepsilon_{\mathrm{air}}^{-0.65}, \tag{4}$$

where $\Pi_g = (p_v - p_{ls})/p_{va}$; $\varepsilon_{air} = p_{air}/p_{va}$; and Nu_{DI} is the diffusion Nusselt number for limitingly low vapor content in the mixture, where the mass-transfer empirical formulas obtained to describe the heat-transfer process can be used. We may use Eq. (2) or Eq. (3) as such formulas, substituting in place of the Prandtl and Nusselt numbers the corresponding diffusion numbers:

$$\mathrm{Pr}_D = \frac{\nu_{va}}{DR_v T_{va}} \text{ and } \mathrm{Nu}_D = \frac{\beta_p d}{D_p}.$$

There exist other descriptions of the boundary conditions, considering mass transfer on the droplet free surface. They do not differ in principle from Eq. (4), the significant advantage of which is that it has been obtained from experiments characteristic of steam turbine condenser operation.

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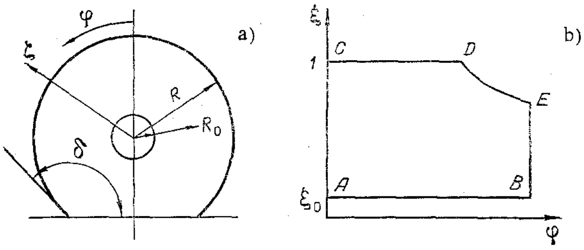

Fig. 1. For solution of droplet growth problem.

As an initial condition we can take $\vartheta(\zeta, \varphi) = 0$ and a prespecified droplet radius, equal, e.g., to the radius of a nucleus, determined by one or the other method.

We will seek the temperature distribution within the droplet by solving the heat-transfer equation:

$$\frac{\partial\vartheta}{\partial t} = a\left(\frac{\partial^2\vartheta}{\partial\zeta^2} + \frac{2}{\zeta}\frac{\partial\vartheta}{\partial\zeta} + \frac{1}{\zeta^2}\frac{\partial^2\vartheta}{\partial\varphi^2} + \frac{\operatorname{ctg}\varphi}{\zeta^2}\frac{\partial\vartheta}{\partial\varphi}\right) \tag{5}$$

for the given boundary and initial conditions. Using the relationships $\vartheta = \theta\vartheta_0$, $t = \tau\tau_0$, $\zeta = \xi R$, we reduce Eq. (5) to the form

$$\frac{\partial\theta}{\partial\tau} = \varkappa\left[\frac{\partial^2\theta}{\partial\xi^2} + \left(\frac{2}{\xi} + \frac{\xi}{a\tau_0}R\frac{\partial R}{\partial\tau}\right)\frac{\partial\theta}{\partial\xi} + \frac{1}{\xi^2}\frac{\partial^2\theta}{\partial\varphi^2} + \frac{\operatorname{ctg}\varphi}{\xi^2}\frac{\partial\theta}{\partial\varphi}\right], \tag{6}$$

which now has an integration range which does not change with time. The coefficient $\varkappa = a\tau_0/R^2$ is naturally time dependent.

We will now introduce an assumption which significantly eases numerical solution of the problem while producing practically no distortion of the physical essence of the phenomenon. We will assume that the droplet center contains a thermally insulated sphere of as small a diameter as desired. Then the half of the droplet in the shape of a truncated sphere (Fig. 1a) in the Cartesian coordinate system ξ, φ takes on the form depicted in Fig. 1b. Now the boundary conditions will be $\theta = 0$ on the line DE, condition (1) at $\xi = 1$, $\partial\theta/\partial\xi = 0$ for $\xi = \xi_0$, $\partial\theta/\partial\varphi = 0$ for $\varphi = 0$ and $\varphi = \pi$. Here $\xi_0 = R_0/R$; R_0 is the radius of the small insulated sphere.

We write Eq. (6) in the form

$$\frac{1}{\varkappa}\frac{\partial\theta}{\partial\tau} = L_1\theta + L_2\theta, \tag{7}$$

where

$$L_1\theta = \frac{\partial^2\theta}{\partial\xi^2} + \left(\frac{2}{\xi}\frac{\partial\theta}{\partial\xi} + \frac{\xi}{a\tau_0}R\frac{\partial R}{\partial\tau}\right)\frac{\partial\theta}{\partial\xi}, \quad L_2\theta = \frac{1}{\xi^2}\frac{\partial^2\theta}{\partial\varphi^2} + \frac{\operatorname{ctg}\varphi}{\xi^2}\frac{\partial\theta}{\partial\varphi}.$$

In the range of continuous change of the arguments ξ and φ we construct a uniform grid with steps $\Delta\xi$ and $\Delta\varphi$. Then the boundary DE of the region of Fig. 1b changes along a steplike line, at the points of which $\theta = 0$.

We replace the operators L_1 and L_2 with difference operators

$$\Lambda_1\theta = \frac{1}{\Delta\xi^2}(\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}) + \left(\frac{1}{\xi_{i,j}} + \frac{\xi_{i,j}}{2a\tau_0}R\frac{\partial R}{\partial\tau}\right)\frac{\theta_{i+1,j} - \theta_{i-1,j}}{\Delta\xi},$$

$$\Lambda_2\theta = \frac{1}{\xi_{i,j}^2\Delta\varphi^2}(\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}) + \frac{\operatorname{ctg}\varphi_{i,j}}{2\Delta\varphi\xi_{i,j}^2}(\theta_{i,j+1} - \theta_{i,j-1}),$$

$$i = 0,\ 1,\ 2,\ \ldots,\ N(\varphi),\quad j = 0,\ 1,\ 2,\ \ldots,\ M(\xi).$$

According to the method of variable directions, the transition from the n-th moment of time to the n+1-th is accomplished in two steps with a time step $\Delta\tau/2$. Initially we solve the equation

$$\frac{2}{\Delta\tau\varkappa}\left(\theta^{n+\frac{1}{2}} - \theta^n\right) = \Lambda_1\theta^{n+\frac{1}{2}} + \Lambda_2\theta^n, \tag{8}$$

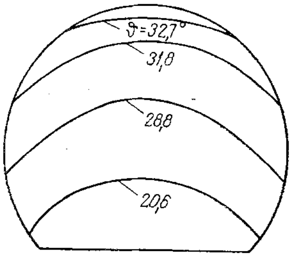

Fig. 2. Droplet temperature head distribution (ϑ, deg) relative to wall temperature.

explicit in φ and implicit in ξ, and then

$$\frac{2}{\Delta\tau\varkappa}\left(\theta^{n+1}-\theta^{n+\frac{1}{2}}\right)=\Lambda_1\theta^{n+\frac{1}{2}}+\Lambda_2\theta^{n+1}, \tag{9}$$

explicit in ξ and implicit in φ. Here θ^n, $\theta^{n+1/2}$ and θ^{n+1} are the values of the desired grid function at the times $\tau=\tau_n$, $\tau=\tau_n+\Delta\tau/2$ and $\tau=\tau_n+\Delta\tau$, respectively.

The coefficient $\varkappa=a\tau_0/R^2$ is defined from the equation

$$r\rho_l\frac{dV}{dt}+\int\limits_{F_{ls}}\alpha(\vartheta_V-\vartheta)\,dF_{ls}=-\lambda_l\int\limits_{F_{ls}}\left.\frac{\partial\vartheta}{\partial\zeta}\right|_{\zeta=R}dF_{ls}, \tag{10}$$

where

$$\frac{dV}{dt}=k\pi R^2\frac{dR}{dt};\;\; k=4-(1-\cos\psi)^2(2+\cos\psi);\;\; \psi=\pi-\delta. \tag{11}$$

Substituting Eq. (11) in Eq. (10), we find

$$\frac{dR}{dt}=-\frac{2}{r\rho_l k}\int\limits_0^\delta\left.\left[\frac{\lambda_l}{R}\frac{\partial\vartheta}{\partial\xi}+\alpha(\vartheta_V-\vartheta)\right]\right|_{\zeta=R}\sin\varphi d\varphi.$$

Solving this equation at each semistep, we find the droplet radius, and hence, the coefficient $\varkappa$. The values of $\partial\vartheta/\partial\xi$ and ϑ on the droplet free surface are then defined from the results for the preceeding moment in time.

With consideration of Eq. (7), Eqs. (8), (9) reduce to the form

$$a_{i,j}\theta_{i+1,j}^{n+\frac{1}{2}}-c_{i,j}\theta_{i,j}^{n+\frac{1}{2}}+b_{i,j}\theta_{i-1,j}^{n+\frac{1}{2}}=-f_{i,j}^n, \tag{12}$$

$$A_{i,j}\theta_{i,j+1}^{n+1}-C_{i,j}\theta_{i,j}^{n+1}+B_{i,j}\theta_{i,j-1}^{n+1}=-F_{i,j}^{n+\frac{1}{2}}, \tag{13}$$

where

$$a_{i,j}=\frac{1}{\Delta\xi^2}+\frac{1}{\Delta\xi}\left(\frac{1}{\xi_{i,j}}+\frac{\xi_{i,j}}{2a\tau_0}R\frac{\partial R}{\partial\tau}\right);\;\; b_{i,j}=\frac{1}{\Delta\xi^2}-\frac{1}{\Delta\xi}\left(\frac{1}{\xi_{i,j}}+\frac{\xi_{i,j}}{2a\tau_0}R\frac{\partial R}{\partial\tau}\right);$$

$$c_{i,j}=\frac{2}{\Delta\xi^2}+\frac{2}{\varkappa\Delta\tau};\;\; f_{i,j}^n=\frac{1}{\xi_{i,j}^2}\left[(\theta_{i,j+1}^n-2\theta_{i,j}^n+\theta_{i,j-1}^n)\frac{1}{\Delta\varphi^2}+\frac{\operatorname{ctg}\varphi_{i,j}}{2\Delta\varphi}(\theta_{i,j+1}^n-\theta_{i,j-1}^n)\right]+\frac{2}{\varkappa\Delta\tau}\theta_{i,j}^n;$$

$$A_{i,j}=\frac{1}{\xi_{i,j}^2\Delta\varphi^2}+\frac{\operatorname{ctg}\varphi_{i,j}}{2\Delta\varphi};\;\; B_{i,j}=\frac{1}{\xi_{i,j}^2\Delta\varphi^2}-\frac{\operatorname{ctg}\varphi_{i,j}}{2\Delta\varphi};\;\; C_{i,j}=2\left(\frac{1}{\xi_{i,j}^2\Delta\varphi^2}+\frac{1}{\varkappa\Delta\tau}\right);$$

$$F_{i,j}^{n+\frac{1}{2}}=\frac{1}{\Delta\xi^2}\left(\theta_{i+1,j}^{n+\frac{1}{2}}-2\theta_{i,j}^{n+\frac{1}{2}}+\theta_{i+1,j}^{n+\frac{1}{2}}\right)+\frac{1}{\Delta\xi}\left(\frac{1}{\xi_{i,j}}+\frac{\xi_{i,j}}{2a\tau_0}R\frac{\partial R}{\partial\tau}\right)\left(\theta_{i+1,j}^{n+\frac{1}{2}}-\theta_{i-1,j}^{n+\frac{1}{2}}\right)+\frac{2}{\varkappa\Delta\tau}\theta_{i,j}^{n+\frac{1}{2}}.$$

We will solve Eqs. (12), (13) by the drive method [8] along the coordinates ξ and φ, respectively. The solution of Eq. (12) will have the form

$$\theta_{i,j}^{n+\frac{1}{2}}=\gamma_{i+1}\theta_{i+1,j}^{n+\frac{1}{2}}+\eta_{i+1,j} \tag{14}$$

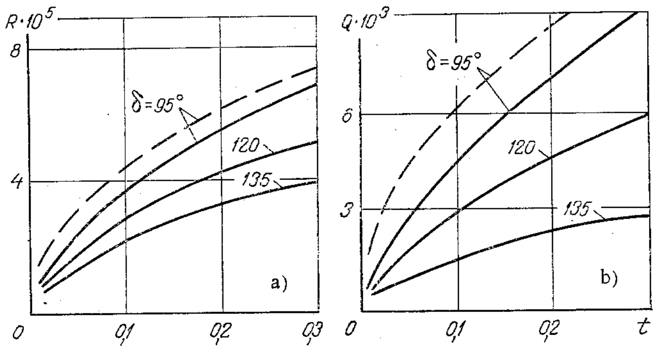

Fig. 3. Droplet radius (R, m) (a) and amount of heat transferred through droplet (Q, W) (b) vs time (t, sec).

where

$$\gamma_{i+1,j}=\frac{a_{i,j}}{c_{i,j}-b_{i,j}\gamma_{i,j}},\quad \eta_{i+1,j}=\frac{f^n_{i,j}+b_{i,j}\eta_{i,j}}{c_{i,j}-b_{i,j}\gamma_{i,j}}. \tag{15}$$

Using the boundary condition $\left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=\xi_0}=0$, we find

$$\gamma_{1,j}=\frac{a_{0,j}+b_{0,j}}{c_{0,j}},\quad \eta_{1,j}=\frac{f_{0,j}}{c_{0,j}}.$$

The remaining values of the drive coefficients $\gamma_{i,j}$ and $\eta_{i,j}$ are found from the recursive relationships (15). Knowing the drive coefficients, we use Eq. (14) to define $\theta^{n+1/2}_{i,j}$. It is then necessary to initially define $\theta^{n+1/2}_{N,j}$ at the droplet surface. This value of the dimensionless temperature is found from boundary condition (1) and Eq. (14)

$$\theta^{n+\frac{1}{2}}_{N,j}=\begin{cases}\dfrac{\eta_{N,j}+\dfrac{\vartheta_v\Delta\xi R\alpha}{\vartheta_0\lambda_l}+\dfrac{\Delta\xi Rr\beta_p}{\vartheta_0\lambda_l}(p_v-p_{1s})}{1-\gamma_{N,j}+\dfrac{\Delta\xi R\alpha}{\lambda_l}} & \text{for}\quad 0\leqslant\varphi\leqslant\delta,\\ 0 & \text{for}\quad \delta<\varphi\leqslant\pi.\end{cases} \tag{16}$$

The equations of Eq. (16) are nonlinear, since β_p and p_{ls} depend upon the droplet surface temperature. The partial pressure of the vapor near the droplet surface then obeys the well-known law of medium pressure as a function of saturation temperature. This function is usually presented tabularly. In the present study the tabular values were approximated by a piecewise linear function at 10° intervals over the range 0-100°C.

The search for the quantity $\theta^{n+1/2}_{N,j}$ over the interval $0\leq\varphi\leq\delta$ was performed in the following manner. Equation (16) was written in the form

$$\Phi\left(\theta^{n+\frac{1}{2}}_{N,j}\right)=\frac{\eta_{N,j}+\dfrac{\vartheta_v\Delta\xi R\alpha}{\vartheta_0\lambda_l}+\dfrac{\Delta\xi Rr\beta_p}{\vartheta_0\lambda_l}(p_v-p_{1s})}{1-\gamma_{N,j}+\dfrac{\Delta\xi R\alpha}{\lambda_l}}-\theta^{n+\frac{1}{2}}_{N,j}=0. \tag{17}$$

Assuming initially that $p_{ls}=p_v$, we find from the equation $p=f(T_{sat})$ the value of T_{sat} and corresponding value of θ. Substituting this value in Eq. (17) we determine the sign of the function Φ. We then reduce the value of $\theta^{n+1/2}_{N,j}$ by an amount $\Delta\theta$ until the sign of the function Φ does not change, thus delimiting the region in which the root of Eq. (17) is located. The refinement is carried further by dividing the interval in half. Equation (13) is solved in a similar manner.

The method described can be used as a basis for solution of the quasistationary problem, where instead of Eq. (6) we solve the equation

$$\frac{\partial\theta}{\partial\bar{\tau}}=\frac{\partial^2\theta}{\partial\xi^2}+\frac{2}{\xi}\frac{\partial\theta}{\partial\xi}+\frac{1}{\xi^2}\frac{\partial^2\theta}{\partial\varphi^2}+\frac{\operatorname{ctg}\varphi}{\xi^2}\frac{\partial\theta}{\partial\varphi}, \tag{18}$$

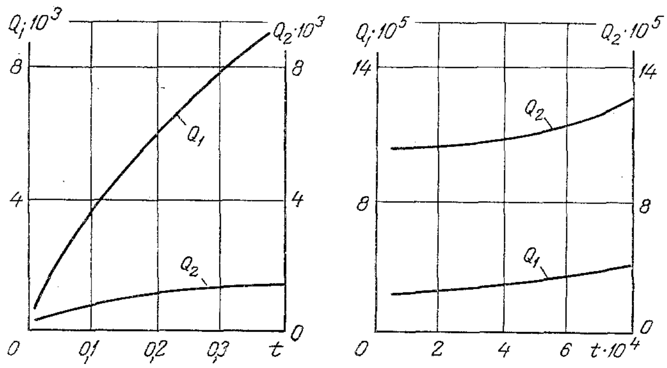

Fig. 4. Quantity of heat removed through droplet by convection (Q_2, W) and condensation (Q_1, W) vs time.

where $\bar{\tau}$ is a fictitious time value. Equations (2) and (16) are calculated for fixed droplet radius. As is well known [9], the solution of the nonstationary equation (18) as $\bar{\tau} \to \infty$ and for stationary boundary conditions tends to the solution of the stationary problem $\Delta\theta = 0$ (where Δ is the Laplacian in the spherical coordinate system).

It is important to note that the solution of the nonstationary problem, which describes the phenomenon more correctly, is less cumbersome and requires less machine time than the solution of quasistationary problems for a series of fixed radius values.

Figure 2 shows the temperature distribution at time $0.5 \cdot 10^{-3}$ sec in a droplet with limiting wetting angle of 135°. Figure 3 shows droplet radius and the quantity of heat transferred through the droplet as functions of times for various wetting angles. Also shown are results of solving the quasistationary problem (dashed lines). We see that consideration of the nonstationary nature of the process has an especially marked effect on calculation of the heat transferred through the droplet (Fig. 3b). With increase in the wetting angle the amount of heat transferred through the droplet decreases. Such a result, although completely understandable, since with increase in this angle the thermal resistance of the droplet increases, still provides no basis to assume that hydrophobic materials providing relatively small wetting angles would be preferable. In fact, it is important for intensification of the process that droplets separate, which naturally occurs more rapidly at larger wetting angles. The simultaneous solution of these two problems will establish the optimum value of wetting angle and permit formulation of the basic requirements relative to hydrophobic materials.

Figure 4 depicts the quantity of heat removed by convection and condensation at a wetting angle $\delta = 95°$. We see that at the beginning of the process, the convective fraction is greater than the condensation, and it is only at sufficiently large droplet size that the generally recognized concept of insignificant convective heat exchange with respect to heat produced by vapor condensation becomes valid.

The results obtained are insufficient to develop an engineering technique of calculating heat exchangers operating in the droplet condensation mode. Such a technique must contain a refinement of the boundary conditions, a solution for droplet coalescence, etc. Nevertheless, the results do provide that part of the technique related to droplet growth.

The study performed permits the conclusion that it is desirable to consider the nonstationary nature of the droplet growth process and the necessity of finding the optimum wetting angle value. It has been established that in calculating the beginning of the droplet condensation process, convective heat exchange cannot be neglected.

NOTATION

$Re = 2wR/\nu_{va}$, Reynolds number; $Pr = \nu/a$, Prandtl number; $Nu = 2\alpha R/\lambda$, Nusselt number; w, flow velocity; R, F, V, radius, free surface, and volume of droplet; δ, limiting wetting angle; d, diameter of condenser tube; ν, λ, a, coefficients of kinematic viscosity, thermal conductivity, and thermal diffusivity; β_p, D, coefficients of mass liberation and diffusion; r, latent heat of vapor formation; R_v, gas constant of vapor; p, partial pressure; ρ, density; ζ, φ, coordinates; t, time; t_0, ϑ_0, characteristic time and temperature. Subscripts: l, liquid; s, saturation; v, vapor; g, air; va, vapor–air mixture; ls, free liquid surface of droplet; nar, narrow section.

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SELF-EXCITATION OF VORTEX STREET INTENSITY BEHIND A PLANE MODEL WITH A BODY IN ITS WAKE

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The study discovered critical conditions under which there is a periodic increase in the Karman vortex street intensity and an increase in pressure pulsations with a change in the distance between a plane model and a body in its wake.

The effect of various external forces on the process of vortex formation behind a model was investigated in [1-3]. The interest in such studies stems from the need to suppress vortex formation in order to reduce the drag of poorly-streamlined bodies.

Similar investigations of the effect of splitter plates on vortex formation in the wake of a plane model were conducted in [4]. The dimensionless frequency of convergence of the vortex without the plate was Sh = 0.24. With an increase in the length of the plate, the dimensionless frequency increased until the ratio of the plate length to half the thickness of the trailing edge $\bar{l} = 1.5$. Here, the value of Sh began to drop sharply; no vortex formation was observed at $\bar{l} = 2.5$, and the pressure coefficient increased. The effect of a splitter plate affixed behind a cylinder along the wake axis on the frequency characteristics of the wake and the resistance coefficient was studied in [2]. It was observed experimentally that the location of a splitter plate behind the cylinder on the wake axis stops vortex formation and lowers the resistance of the cylinder by about 50%. It was shown that the base pressure increases when the plate is moved away from the cylinder to a distance equal to about four cylinder diameters, and that this is accompanied by a reduction in vortex convergence frequency. Any large displacement of the plate is accompanied by a sharp increase in the vortex convergence frequency, to nearly its initial value, and a corresponding sharp decrease in base pressure.

It was shown in [5, 6] that the formation and separation of discrete vortices behind a plane model are accompanied by pressure pulsations near the trailing edge and the propagation of density waves in the flow at a frequency equal to the vortex convergence frequency. In [7] a study was made of the wave intensity and length, the directionality of the vortex sound, and the propagation of sound waves radiated by the vortex street. Also examined was the nature of interaction of the pressure pulsations in the flow and the sound waves radiated by a Karman street. Pressure pulsations and sound waves generated by mediums which are similar in spectral composition but physically quite different may interact, changing the pressure in the region of vortex formation and the intensity of the Karman vortex street.

In the present work, we studied the features of vortex formation with a change in pressure in the region of vortex formation behind a plane model with a body located in its wake. We also studied the effect of pressure pulsations in the wake on the intensity of the vortex formation process.

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